Principles of optics

Electromagnetic theory of propagation, interference and diffraction of light

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